DYNAMIC RADIATION COLLAPSE

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Self-similar compression regimes of a finite plasma mass are considered with allowance for intensive radiation loss.

Researchers have long been interested in formulating the problem of compression of a finite plasma mass [1-3]. It is of interest to consider a similar problem for the compression of an intensively radiating gas, heated to high temperatures, e.g., when high-velocity plasma bunches collide. Such high-velocity bunches can be produced by the action of laser pulses or electron beams on a foil or by electrodynamic acceleration in explosive or magnetoplasma compressors. As they fly apart the density of the bunches gradually decreases and may be rather low by the time of collision. In this case only part of the emitted radiation is absorbed while the energy loss of a unit mass per unit time from the collision region is quasivolumetric and can be written as $Q = 4\kappa\sigma T^4$, where κ is the effective mass coefficient of emission with allowance for self-absorption, corresponding to the blackness η of the entire plasma bunch and the dimension r_0 at the given temperature [4].

Intensive radiation cools the gas and reduces the pressure in it, which will result in the formation of a more compact, denser bunch than in the absence of radiation loss.

A well known way of compressing is by utilizing the pressure of a magnetic field generated by some external system or the intrinsic electron current flowing in the plasma (in Z-pinch geometry). The compression of a cylindrical column of radiating plasma heated by a current has been considered in many studies [5-8]. The current causes the column of the compressed plasma to heat up. In the case of "fast" systems, however, because of the skin effect only the outermost layers of the plasma are heated while the remainder is accelerated by magnetic piston action. The effect of the magnetic field can be ignored when the pulse in the implosion stage is short. Radiation collapse occurs under comparatively long pulses of electric current, when radiating Z pinch undergoes quasisteady compression [9, 10]. It is of interest to consider the problem of an essentially transient dynamic radiation collapse.

Another way of compressing a plasma is by means of ablation of its outer layers as a result of laser radiation or a beam of fast particles. "Burning" under the effect of the laser pulse or particle beams and flying outwards, the plasma acts as a piston, compressing the "core." If the elements of the sheath have a low atomic number, the sheath itself radiates only weakly. If the gas being compressed consists of heavy elements and is preheated in some way and the temperature of the plasma "core" is high, it will emit rather hard radiation that freely passes through the sheath. If the laser pulse is short relative to the compression and radiation time of the "core," its role is reduced to merely "imparting" the initial velocities. The density of the "core" can be expected to increase considerably when the radiation is intensive. Without specifying how the plasma is compressed and how it is preheated, we can consider the problem of it being compressed when it is given initial, inwardly directed velocities.

We write the energy equation with allowance for the volumetric or quasivolumetric radiation loss:

$$\frac{\partial e}{\partial t} + p\frac{\partial v}{\partial t} = -Q(e, v). \tag{1}$$

Suppose that the loss rate Q has a power-law dependence on the thermodynamic parameters:

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$$Q = Q_* (e/e_*)^{-\alpha} (v/v_*)^{-\beta} \varphi(m/m_*). \tag{2}$$

The introduction of the function $\varphi(m/m_*)$ makes it possible to consider the existence of many layers with different optical properties. For example, $\varphi = 0$ for a nonradiating sheath of light elements while we can set $\varphi = 1$ for the "core." The equation of state can be written as

$$e = pv/(\gamma - 1). \tag{3}$$

For aluminum in the temperature range T = 10-120 eV we can assume that $\kappa \sim T^{-3}v^{-2/3}$ [4], $e \sim T^{3/2}$, whereby $\alpha = -2/3$, $\beta = 2/3$. In this range $\gamma = 1, 2$. The quantities e_*, v_*, and m_* are some characteristic values.

Let us consider one-dimensional (planar, cylindrical, spherical, $\nu = 1, 2, 3$) gas motions. System (1)-(3) is supplemented by the equations of motion and continuity

$$\frac{\partial u}{\partial t} + r^{\nu-1} \frac{\partial p}{\partial m} = 0, \quad \frac{\partial r}{\partial t} = u, \quad \frac{1}{\nu} \frac{\partial r^\nu}{\partial m} = v. \tag{4}$$

System (1)-(4) can be reduced to dimensionless form by the transformation

$$u = u_* u', \ v = v_* v', \ e = e_* e', \\ t = t_* t', \ p = p_* p', \ m = m_* m', \ r = r_* r' \tag{5}$$

(the primes here denote dimensionless quantities).

Only three of the dimensional quantities are independent. When m_* (the total gas mass), u_* (the characteristic initial gas velocity), and v_* (characteristic initial specific volume) are chosen to be the independent quantities the other dimensional quantities are determined from the relations

$$e_* = u_*^2, \ r_*^\nu = m_* v_*, \ t_* = r_*/u_*, \ p_* = u_*^2/v_*. \tag{6}$$

We should point out that e_* is the characteristic internal-energy value that would have been attained if the entire kinetic energy were transformed into thermal energy in the absence of radiation; r_* is the characteristic initial dimension; and t_* is the characteristic compression time without allowance for the retarding effect of the gas motion. All of the equations, except the energy equation, are invariant under transformation (5). When the primes of the dimensionless variables are omitted, the energy equation can be written as

$$\frac{\partial e}{\partial t} + p \frac{\partial v}{\partial t} = -\tau e^{-\alpha} v^{-\beta} \varphi. \tag{7}$$

The dimensionless parameter τ is determined by the relation

$$\tau = \frac{Q_* t_*}{e_*} = \frac{Q_* t_*}{u_*^2}. \tag{8}$$

It represents the ratio of the characteristic time t_g of gasdynamic motion to the de-excitation (radiation) time $t_r = e_*/Q_*$. Let us consider a specific example. Suppose that aluminum-plasma bunches of dimension r_0 = 0.3 mm and density $\rho \sim 3 \cdot 10^{-3}$ g/cm³ are compressed. At a temperature T ~ 120 eV the blackness is $\eta \sim 2 \cdot 10^{-3}$ [4]. At an anergy $e \sim 10^4$ kJ/g the radiation time is $t_r \sim 6$ nsec. When the velocity is $u = 1.4 \cdot 10^7$ cm/sec the characteristic gasdynamic time is $t_g \sim 2$ nsec, i.e., $\tau \sim 0.3$. The radiation loss will substantially affect the gas dynamic motion.

In the general case system (3), (4), (7) must be solved numerically. Shock waves, which can arise inside the compressing and radiating volume, propagate toward and away from the center and heat up the gas. There may be situations, however, when the gas is preheated by auxiliary laser beams, current pulses, and so forth. Then shock and acoustic waves rapidly propagate in the plasma, causing a weakly varying parameter profile to be established. Let us consider the limiting shockfree regimes of compression and radiation by a preheated plasma. For simplicity we confine the discussion to compression of a plasma with homogeneous properties, i.e., we set $\varphi \equiv 1$.

We assume that motion with radiation loss occurs in a so-called regular regime and we look for the solution of the system in separable variables [11-15]:

$$r = R(m)\,r^0(t),\ \ p = P(m)\,p^0(t),$$

$$v = V(m)\,v^0(t),\ \ u = U(m)\,u^0(t). \tag{9}$$

Upon substituting (9) into Eq. (7) and the equations of motion, we obtain two systems of ordinary differential equations, one of which is used to find the time dependence of the parameters,

$$\frac{de^0}{dt} + p^0\frac{dv^0}{dt} = -\tau C_e (e^0)^{-\alpha}(v^0)^{-\beta},\ \ \frac{1}{u^0}\frac{dr^0}{dt} = C_r,$$

$$\frac{du^0}{dt} = C_u p^0 (r^0)^{\nu-1},\ \ v^0 = C_v (r^0)^{\nu},\ \ e^0 = \frac{p^0 v^0}{\gamma - 1}\ , \tag{10}$$

and the other is used to determine the mass distribution of the parameters,

$$(PV)^{-(\alpha+1)}V^{-\beta} = C_e,\ U = C_r R,\ R^{\nu-1}dR = C_v V dm,\ -R^{\nu-1}dP = C_u U dm. \tag{11}$$

The symbols C_i in Eqs. (10) and (11) are separating constants. By a method similar to that in [15] we can show that only one combination of these constants has the connotation of a scale factor and it can be introduced into the definition of the scale of the quantity. With the choice $C_u = C_r = C_v = C_e = 1$ the first of Eqs. (11) is reduced to the form $PV^{\xi} = 1$, where $\xi = \beta/(\alpha + 1) + 1$. This system of equations is solved with the following boundary conditions:

m = 0, U(0) = 0, and R(0) = 0 at the inner boundary by virtue of the symmetry;

m = 1 and P(1) = 0 at the inner boundary for compression with a preassigned velocity.

The solution of system (11) is in fact analogous to that given in [11-13] for the problem of plasma spread with heating. It is completely solved analytically for $\nu = 2$ but must be solved numerically for other cases.

Let us analyze the time dependence of the parameters, omitting the subscript 0. The corresponding system has the form

$$\frac{de}{dt} + p\frac{dv}{dt} = -\tau e^{-\alpha}v^{-\beta},\ \ \frac{dr}{dt} = u,\ \ \frac{du}{dt} = pr^{\nu-1},\ \ v = r^{\nu}. \tag{12}$$

For special initial and boundary conditions a solution as a power function of time exists in the form $(t_f - t)^s$, where t_f is the time of compression to infinite density.

$$r = A(t_f - t)^s,\ \ u = -As(t_f - t)^{s-1},\ \ v = A^{\nu}(t_f - t)^{s\nu},$$
$$e = B(t_f - t)^k,\ \ p = (\gamma - 1)B(t_f - t)^{k-\nu s}/A^{\nu}. \tag{13}$$

Two relations for the exponents result when Eq. (13) is substituted into Eq. (12):

$$k = 2s - 2,\ \ s = (3 + 2\alpha)/(2(1+\alpha) + \nu\beta). \tag{14}$$

For chosen α and β at each value of ν such a solution can exist only at one particular value of s. Moreover, two equations are obtained for the amplitudes:

$$A^2 s(s-1) = (\gamma - 1)B,\ \ B(k + \nu s(\gamma - 1)) = \tau B^{-\alpha}A^{-\nu\beta}. \tag{15}$$

From (15) we get two constraints for the exponent s:

$$s > 2/(2 + \nu(\gamma - 1)),\ \ s > 1. \tag{16}$$

The first inequality of (16) indicates that compression with radiation losses should occur more rapidly than adiabatic compression since $s_* = 2/(2 + \nu(\gamma - 1))$ corresponds to adiabatic compression [2]. The second requires retardation as the compression proceeds. At a given choice of α and β these conditions are satisfied only for $\nu = 1$.

For the problem of "falling-in" (collapse) under consideration here we can take $u(t_0) = u_0 = -1$. Without loss of generality we can find $t_0 = 0$. Moreover, with the requirement that $t_f = 1$ we obtain

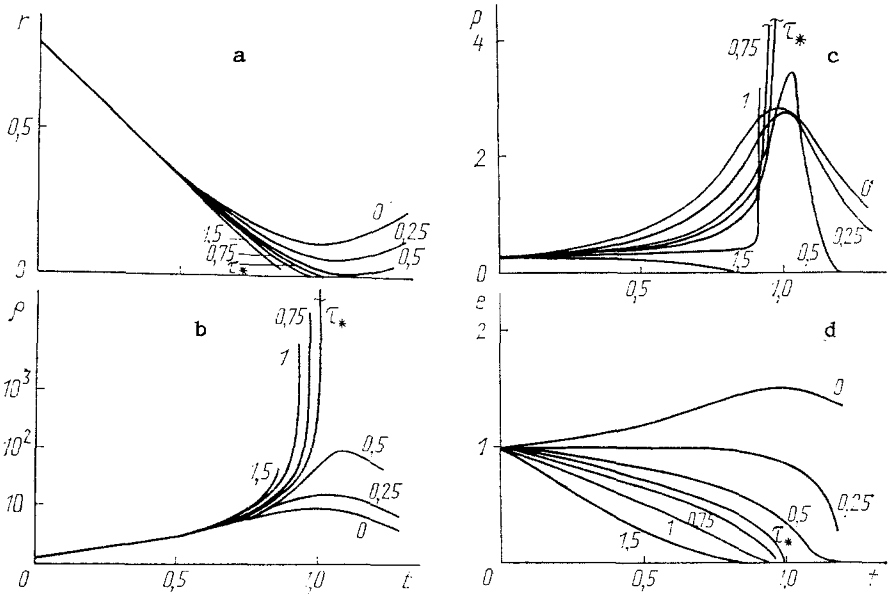

Fig. 1. Radius r(a), density ρ(b), pressure p(c) and internal energy e(d) vs. time at different values of τ in the planar case (ν = 1).

$$A=\frac{1}{s},\quad B=\frac{\gamma-1}{s-1}\,\frac{1}{s},\quad \tau=B^{1+\alpha}A^{\nu\beta}\,(k+\nu s(\gamma-1)). \tag{17}$$

A power solution at $u_0 = -1$, $t_f = 1$ is possible only for a single value $\tau = \tau_*$. With the specific choice of parameters $\alpha = -2/3$, $\beta = 2/3$ for $\nu = 1$ the quantities vary as follows as the collapse time t_f is approached: $u \sim (t_f - t)^{1/4} \to 0$, i.e., the plasma is slowed down and cools since $e \sim (t_f - t)^{1/2} \to 0$. The density increases more rapidly than does the pressure:

$$p\sim(t_f-t)^{-3/4}\to\infty,\quad v\sim(t_f-t)^{5/4}\to 0.$$

The results of numerical calculations of system (12) for the planar case ($\nu = 1$) at different values of the parameter τ are given in Fig. 1. The graphs of the radius r, density $\rho = 1/v$, pressure p, and internal energy e as functions of time are shown in Fig. 1a, b, c, and d, respectively. The values of the parameter τ are given next to the respective curves. The initial values of the radius, velocity, and internal energy $r_0 = 0.8$, $u_0 = -1$, and $e_0 = 1$ satisfy the exact solution at $s = 1.25$ and $\tau_* = 0.643$, $\nu = 1$. As already mentioned, τ characterizes the ratio of the time of gasdynamic motion to the characteristic radiation time t_r. The value $\tau = \tau_*$ separates two regimes. At $\tau < \tau_*$ the compression continues to a time $t > t_f$ (Fig. 1a) the density increases from 1.25 to $\rho \sim$ 10-100 (Fig. 1b). The gas is then stopped completely (u = 0), the pressure reaches its maximum value (Fig. 1c). Compression is replaced by spreading after that.

At $\tau = 0$, i.e., in the absence of radiation loss, the internal energy e at first increases upon retardation and then decreases because of spreading (Fig. 1d). At $\tau \neq 0$ the radiant energy loss either balances the increase in the internal energy upon compression ($\tau \leq 0.25$) or exceeds it ($\tau > 0.25$). When τ increases (at $\tau > \tau^*$) as t_f is approached the gas cools while continuing to move. In this case $r \to 0$ and $\rho \to \infty$ (Fig. 1a, b, d). When $r \to 0$ and $e \to 0$ we can determine which process is faster, the increase in compression or energy loss. if $p \to \infty$ as the final state is approached, $\rho \to \infty$ at a faster rate than $e \to 0$. This is what happens at $\tau = 0.75$ and 1. A further increase in τ (e.g., at $\tau = 1.5$) results in a transition to a regime of energy loss due to radiation without retardation, when in the final state $p \to 0$. The change in the initial state, a decrease in the initial faction of the kinetic energy leads to the fact that compression followed by spreading is immediately replaced by a regime with a dominant radiation energy loss.

Inclusion of radiation in the energy equation, therefore, changes the nature of the evolution of the parameters. The radiation loss causes a much higher degree of compression to be reached.

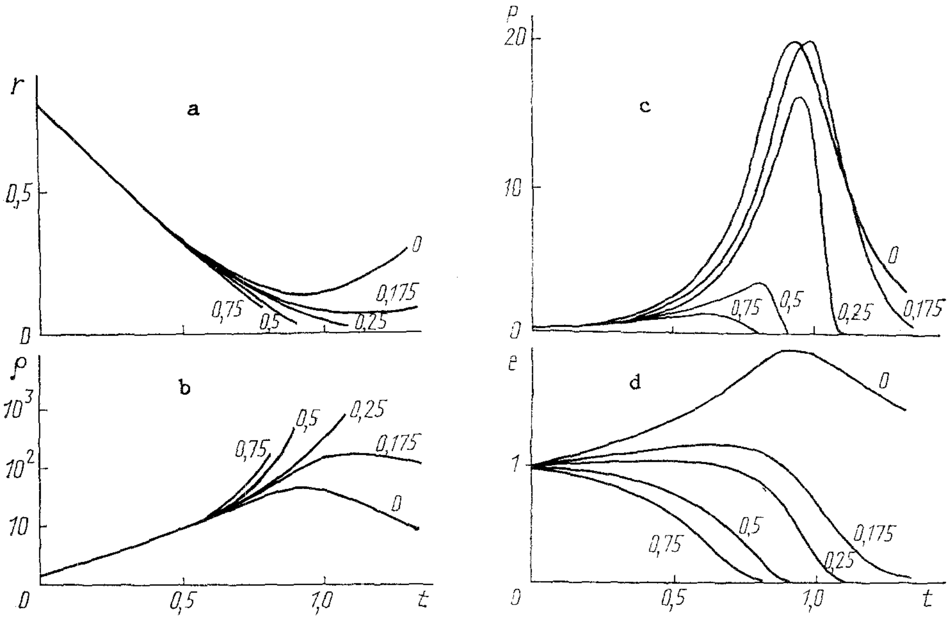

Fig. 2. Radius r (a), density ρ (b), pressure p (c) and internal energy e (d) vs. time at different values of τ in the cylindrical case (ν = 2).

A solution of the form (13) does not exist at $\alpha = -2/3$ and $\beta = 2/3$ for $\nu = 2, 3$. Figure 2 shows the results of numerical calculation for the cylindrical case. Since $\rho \sim 1/r^2$ (and not 1/4 as in the planar case), compression causes a more pronounced increase in the internal energy e and the density ρ and a smaller decrease in the radius at $\tau = 0$ than in the planar case (Fig. 2).

At $\tau = 0.175$ the compression is replaced by spreading and at $\tau \geq 0.25$ by the regime of radiation without retardation. In the final state $e \to 0$, the density $\rho \sim 10^2\text{-}10^3$, and the pressure passes through a maximum before maximum compression and then falls off to zero.

In the case of large values of τ we can assume (especially for $\nu = 2, 3$) that radiation occurs at roughly constant specific volume v_1 and we write the energy equation as

$$\frac{de}{dt} = -\tau e^{-\alpha} v_1^{-\beta} = -\tau_1 e^{-\alpha}, \text{ where } \tau_1 = \tau v_1^{-\beta}.$$

which means that $e^{1+\alpha} = e_i^{1+\alpha}\left(1 - \frac{\tau_1 t(1+\alpha)}{e_0^{1+\alpha}}\right)$, i.e., the energy is radiated in a finite time $t_r = e^{1+\alpha}/(1+\alpha)\tau_1 = 3e_0^{1/3}/\tau_1$. At the final density this causes $p \to 0$. Compression proceeds without stop. We can estimate the time t_r; e.g., for $\nu = 2$ and $\tau = 0.5$ the radiation time is $t_r \sim 0.3$. The increase in the density during compression causes the cooling rate to increase.

In conclusion, we consider some limitations on the formulation of the problem under consideration. First, power-law approximations of the type (2) are valid (even roughly) only in a limited temperature range. At low temperatures the quantity κ and with it the quantity Q decrease rapidly. Cooling actually takes place, therefore, not to T = 0 but only to some value T_{min}, which can be much lower than the initial temperature, and the bulk of the cooling process is described in the aforementioned approximation. A more exact description can be obtained by using more involved dependences Q(e, ρ), e.g., of the form $Q_2(e) \cdot Q_2(\rho)$, where Q_1 increases, starting from some values of e in accordance with the tabulated values of κ [4]. The description can be refined further by using tabulated values of Q(e, ρ), but discarding the self-similar formulation. Second, upon compression to high densities, especially when the temperature falls at the same time, κ can increase so much that the approximation of volumetric or quasi-volumetric approximation of de-excitation ceases to be valid. In this case we must either go over to a different formulation (e.g., with surface energy losses) or solve the direct radiation-gasdynamic problem with the complete radiation transport equation.

NOTATION

Here, T denotes the temperature; σ is the Stefan-Boltzmann constant; η is the blackness coefficient; κ is the effective mass coefficient of emission; ρ is the density; γ is the effective adiabatic exponent; Q is the rate of radiative energy loss per unit mass; Q_* is the dimensional coefficient in the energy-loss law; α and β are exponents in the energy-loss law; u, m, r, v, p. e, and t are the velocity, mass and Eulerian coordinates, specific volume, pressure, internal energy, and time, respectively; * denotes isolated time dependences; ' denotes dimensionless variables; ν is the parameter characterizing the chosen geometry; τ is a parameter introduced into (8); P, V, U, and R are isolated coordinate dependences of the pressure, specific voluem, velocity, and the Eulerian coordinate; t_g and t_r are the characteristic gasdynamic and radiation times; t_0 is the initial time; t_f is the collapse time; C_e, C_r, C_u, and C_v are separating constants; A and B are parameters that appear in the power laws of the time variation of the parameters; and k and s are the exponents in those laws.

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